Slip Angle Accuracy

Subtitle: How and why does slip angle accuracy change with speed?
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1. Introduction

In automotive testing "slip angle" is a term used to describe the ratio of forward and lateral velocities, expressed as an angle. In ISO8855 it is called the sideslip angle and given the symbol β. It is calculated as:

$$\beta = \arctan(Vy/Vx)$$

where Vy and Vx are the lateral (left) and longitudinal (forward) velocities of the vehicle.

It is clear from this equation that you cannot compute slip angle when both velocities are zero and you would need a different form of the equation when Vx is zero. This is important to understanding the problems associated with measuring slip angle at low speeds.

ISO15037 recommends measuring slip angle with a maximum error of ±0.5°, though most manufacturers require a higher accuracy than this. No clear statistical measure is defined in ISO15037 and neither will they be used in this document.

2. Uses of slip angle

Slip angle is used in a variety of objective vehicle dynamics measurements. These include checking the stability of the vehicle, the feel of the steering, comparing the performance of different tyres. One of the most important areas is ESC tuning: because the sensors in vehicles are not able to accurately derive the slip angle, a model is needed to estimate slip angle; this model needs tuning with accurate data.

3. Problems with measuring slip angle

The main problem in measuring slip angle is that it is a very small angle. A change of 0.5° is significant. Let's stop and think about how small a 0.5° angle is. At school a protractor only had markings every 1° and it is pretty difficult to be accurate to better than 1 degree when drawing lines on paper.

Slip angle also varies quickly, so fast changes (above 1Hz) are significant. You cannot average a series of measurements together in order to improve slip angle because you will lose important information in your filter.

Slip angle changes depending on where you measure it in the vehicle. At the front of the vehicle the slip angle (of the car body, not the tyres) is much larger than at the rear. When the tyres turn then the steer angle (δ) directly affects the slip angle at the front of the car. Assuming a wheelbase of 3m and a steer angle of 3°, position where the slip angle is measured needs to be controlled to within 0.5m to achieve 0.5° accuracy, which is not usually too hard. To measure with an accuracy of 0.1° it needs to be controlled to within 0.1m, which is more difficult. This is a particular problem when benchmarking cars where the wheelbase, centre of gravity and driver's seat may be different.

The height in the vehicle is also significant. Roll rates in dynamic tests can be as high as 1rad/s (57°/s). This induces a difference in lateral velocity over a height change of 1m of 1m/s. If the vehicle is travelling at 20m/s (72km/h) then the slip angle will vary by nearly 3 degrees over the 1m height. For 0.5° accuracy it is important to control the height of the measurement point to 0.5m.

The road camber is also a source of error in slip angle measurement. Some vehicles can change slip angle by 0.5° with a road angle change of 2.8°. (Road camber is normally at least 1.4° so rainwater can drain off.) Note that this does not mean that the driver has to adjust, the car suspension may compensate for this by design. Vehicle proving grounds usually have a smaller camber than 1.4°. Crowned roads have a significant effect on slip angle, particularly on data from sine tests.

These problems are independent of the instrument used to measure the slip angle and they are all significant sources of error. One other source of error is the alignment of the instrument so that it is lined up with the car axes. Normally this is compensated in software.

Next we will look at how instruments, in particular navigation instruments, measure slip angle.

4. Slip angle measurement

There are several different ways to measure slip angle but they all divide into two categories: either they measure in the vehicle's co-ordinate frame or they measure in an earth-based co-ordinate frame.

Products that measure in the vehicle's co-ordinate frame include the Corrsys optical sensors. These measure the forward and lateral velocities directly and can compute slip angle using the formula defined in ISO8855. People who use optical sensors will be familiar with the noise increasing at lower speeds and some new Corrsys products use two technologies, one better at low speeds and another better at high speeds.

Products that measure in an earth-based co-ordinate frame include GPS and inertial navigation systems. Hemisphere and Trimble both make specialised GPS products that can measure heading from two GPS antennas. Inertial navigation systems, such as our RT3000 series, can measure heading from one GPS antenna or this can be supplemented by a second GPS antenna for some applications (normally aircraft or boat based).

The basic measurements of a navigation system are:

Linear Based	Angular Based
Acceleration (primary)	Angular acceleration
Velocity	Angular rate (primary)
Position	Orientation

For navigation systems to compute slip angle they have to use measurements of velocity and orientation as follows:

β = heading – atan(Ve/Vn)

where Vn and Ve are the velocity north and velocity east respectively. The atan() function needs to evaluate the angle of Ve and Vn into four quadrants correctly.

Note that heading refers to the direction the vehicle is pointing, and not the direction it is travelling (course over ground). It is common for GPS receivers to use the term heading when track or course over ground should be used, but this is incorrect. The term atan(Ve/Vn) is the track angle or course over ground and it can be computed by single antenna GPS systems.

5. Accuracy of slip angle for navigation systems

Since slip angle, β, depends on two terms, the accuracy of β depends on the accuracy of these two terms. This article is not going to go into the mathematical depths and show the derivations. We will just look at some facts that look obvious when we consider the problem in detail. They can be derived, if necessary.

We can write out:

Accuracy(β) = SomeFunction(Accuracy(heading), Accuracy(atan(Ve/Vn))

or, descriptively, the accuracy of slip angle, β, is some function of the accuracy of heading and the accuracy of atan(Ve/Vn). We will assume, for now, that the accuracy of heading is independent of the accuracy of atan(Ve/Vn) and then we can consider each term separately.

It stands to reason that:

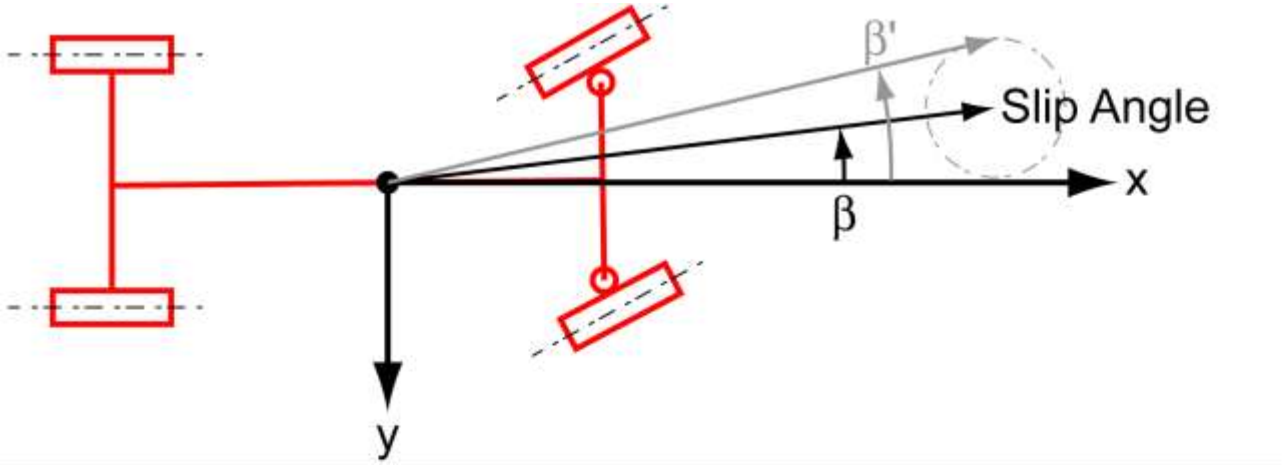
Accuracy(β) $\geq$ Accuracy(heading)

or, descriptively, the accuracy of slip angle is greater than or equal to the accuracy of heading. You cannot measure slip angle more accurately than heading (assuming independence).

If you want to know how accurate an instrument can measure slip angle, you can start by looking at the heading accuracy and know that it will never be better than that. (Or you will need a statement that heading and velocity errors are correlated.)

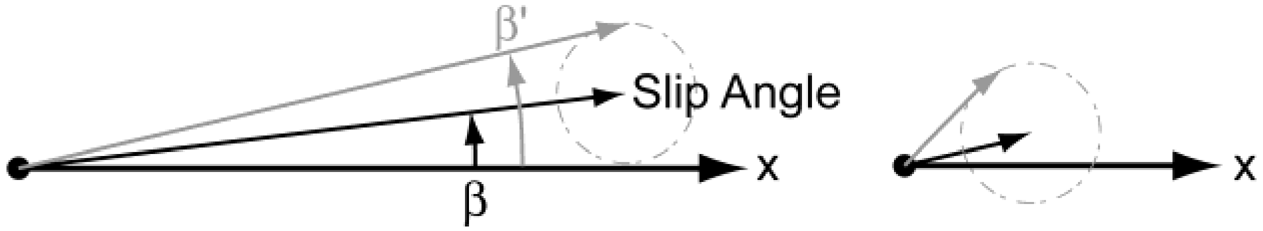
An instrument that can measure heading to an accuracy of 0.1° will not measure slip angle more accurately than 0.1°.

Next we will look at the accuracy of atan(Ve/Vn). The errors can be drawn out as follows:



In this diagram the heading is defined by the x-axis of the vehicle, and we are assuming no error in heading since we have covered this already. The slip angle line is also the course over ground or atan(Ve/Vn). Around this is a grey circle, which represents the error in the velocity measurement. The β' angle is an angle that could be measured when velocity noise is considered. (Normally an instrument measures β' and the true slip angle can be anywhere inside a circle at the end of β', but this is not drawn here.)

The slip angle line is actually the velocity vector and it changes length depending on how fast the vehicle is travelling. For GPS-based velocity systems (including GPS-aided inertial navigation systems) the error circle is relatively constant and is not dependent on speed. The effect on the error can be seen below.



As the velocity line gets shorter (right) and the effect of the error on the slip angle increases; see how the same sized circle has caused the grey slip angle measurement to have a bigger difference compared to the true slip angle (black). The diagram on the right has a much shorter velocity error, but the reported slip angle (grey) has a bigger error on it compared to the diagram on the left.

We can write this as a mathematical function and this approximates to:

Accuracy(atan(Ve/Vn)) = atan(Accuracy(V)/Speed))

Let's explore the effect of this. Assuming Accuracy(V), or accuracy of velocity, is 0.1km/h (typical for GPS after filtering and easily achieved by a GPS-aided inertial navigation system) then we can work out the following table for Accuracy(atan(Ve/Vn)):

Speed (km/h)	Accuracy(atan(Ve/Vn))
10	0.57°
20	0.29°
50	0.11°
100	0.06°
200	0.03°

Similarly to heading:

Accuracy(β) ≥ Accuracy(atan(Ve/Vn))

or, descriptively, the accuracy of slip angle is greater than or equal to the accuracy of atan(Ve/Vn). You cannot measure slip angle more accurately than atan(Ve/Vn) (assuming independence).

Now that we have reduced the problem into two parts, all we need to do is combine the two accuracies together. Here it is important to assume independence and it is important to assume Gaussian distribution of the errors. After several pages of maths (especially if you derive it from first principles) you can arrive at the equation:

Accuracy(β) = sqrt(Accuracy(heading)^2 + Accuracy(atan(Ve/Vn))^2)

or

Accuracy(β) = sqrt(Accuracy(heading)^2 + atan(Accuracy(V)/Speed))^2)

Using the assumptions that heading is accurate to 0.15° and velocity is accurate to 0.1km/h, we can compute the following table for the accuracy of slip angle.

Speed (km/h)	Slip Angle
10	0.59°
20	0.32°
50	0.19°
100	0.16°
200	0.15°

6. Conclusions

The conclusions of the document are the accuracy of slip angle depends on many factors. Vehicle geometry, position of measurement, bandwidth and road geometry all affect the accuracy that can be measured.

For the navigation-based measurement instrument it is unrealistic to expect slip angle to be more accurate than heading. For all measurement instruments it is wrong to assume that slip angle has a constant accuracy. The accuracy depends on speed and gets progressively worse as speed decreases.

In order to know the accuracy of slip angle, it is important to mount the instrument carefully in the car, to know the point where the slip angle should be measured, to check all the specifications of the instrument and to compute the expected slip angle accuracy for every test.